Name solutions

April 12, 2012

ECE 311 Exam 3 Spring 2012

Closed Text and Notes

1) Be sure you have 10 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(10 pts) 1. In a region of space where $\mathbf{E} = 10^3\,\hat{\mathbf{a}}_y \dfrac{\mathrm{V}}{\mathrm{m}}$ and $\mathbf{B} = 0.1\hat{\mathbf{a}}_z$ T an electron moves with a constant velocity. What is the velocity of the electron?

Since the velocity is constant, there is no force acting on the electron
From the Lorentz force equation

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B}) = 0$$

$$\vec{u} \times \vec{B} = -\vec{E}$$

$$\vec{u} \times \left(0.1\,\hat{a}_z \frac{Wb}{m^2}\right) = -10^3\,\hat{a}_y \frac{V}{m}$$

so $\vec{u} = u_o \hat{a}_x$ where $u_o > 0$ in order for $\vec{u} \times \vec{B}$ to be in the $-\hat{a}_y$ direction.

$$u_o \hat{a}_x \times \left(0.1\,\hat{a}_z \frac{Wb}{m^2}\right) = -u_o\left(0.1\frac{Wb}{m^2}\right)\hat{a}_y = -10^3\,\hat{a}_y \frac{V}{m}$$

$$u_o = \frac{10^3 \frac{V}{m}}{0.1 \frac{Wb}{m^2}} = 10^4 \frac{V}{m}\frac{m^2}{Wb} = 10^4 \frac{m}{s} \quad \text{so} \quad \vec{u} = +10^4 a_x \frac{m}{s}$$

(6 pts) 2. Match the material with the descriptions below the table.

| | |
|---|---|
| Paramagnetic | A |
| Ferromagnetic | C |
| Diamagnetic | B |

A. The atoms in the material have a small magnetic dipole moment, the magnetization is in the same direction as the applied field, but the effect is very small.

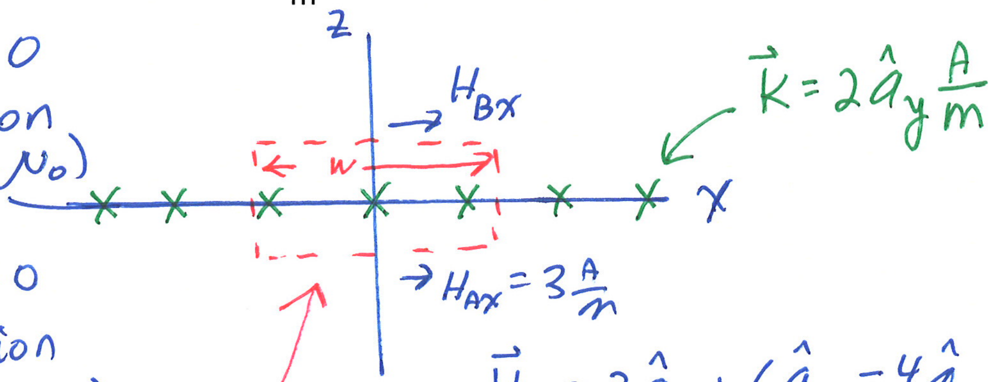
B. The atoms in the material have no magnetic dipole moment, the magnetization is in the opposite direction of the applied field, but the effect is very small.

C. The atoms have a very large magnetic dipole moment, the magnetization is in the same direction as the applied field and the effect is very large.

(9 pts) 3. For $z < 0$, $\mu = 2\mu_o$ and $\mathbf{H} = 3\hat{\mathbf{a}}_x + 6\hat{\mathbf{a}}_y - 4\hat{\mathbf{a}}_z \frac{A}{m}$. On the z = 0 plane there is a sheet current density of $\mathbf{K} = 2\hat{\mathbf{a}}_y \frac{A}{m}$. For $z > 0$ $\mu = \mu_o$. Find $\mathbf{H}$ for $z > 0$.

let $z > 0$ be region B $(\mu_B = \mu_o)$

Let $z < 0$ be region A $(\mu_A = 2\mu_o)$




$$\vec{H}_A = 3\hat{a}_x + 6\hat{a}_y - 4\hat{a}_z \ \frac{A}{m}$$

$$\oint \vec{H}\cdot d\vec{l} = H_{Bx}W - H_{Ax}W = KW$$

$$H_{Bx} = H_{Ax} + K = 3\frac{A}{m} + 2\frac{A}{m} = 5\frac{A}{m}$$

for a similar path perpendicular to the x-axis $I_{enclosed} = 0$ so,

$$H_{By} = H_{Ay} = 6\frac{A}{m}$$

The normal components of the magnetic flux density are continuous across the boundary

$$B_{Bz} = B_{Az} = \mu_A H_{Az} = 2\mu_o\left(-4\frac{A}{m}\right)$$

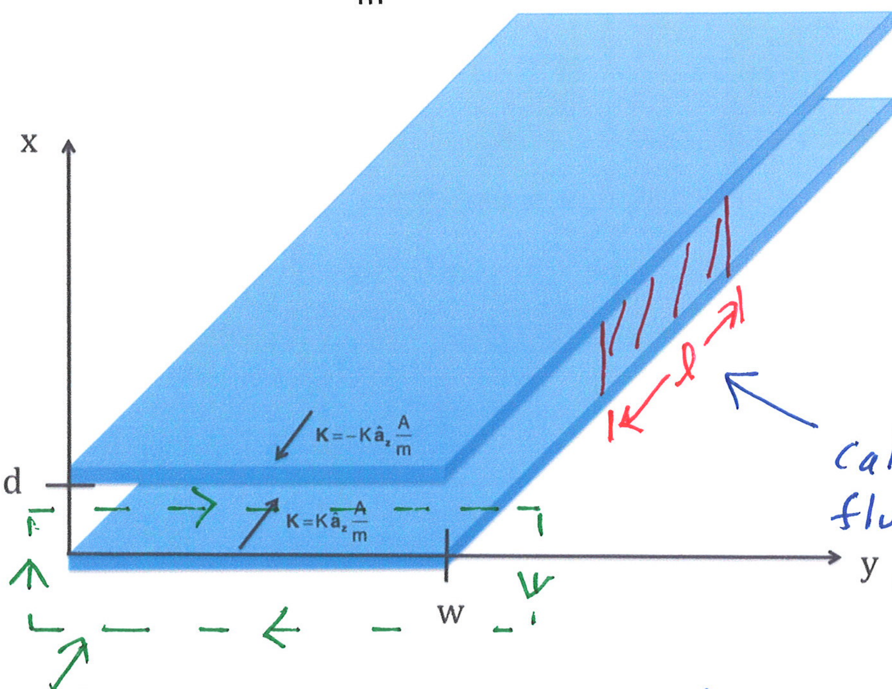
$$B_{Bz} = \mu_B H_{Bz} = \mu_o H_{Bz} = 2\mu_o\left(-4\frac{A}{m}\right)$$

$$H_{Bz} = -8\frac{A}{m}$$

$z > 0$

$$\vec{H}_B = 5\hat{a}_x + 6\hat{a}_y - 8\hat{a}_z \ \frac{A}{m}$$

(15 pts) 4. Find the inductance per unit length for the parallel conductors shown. Ignore fringing fields, which means you can approximate the magnetic field between the plates as if the plates were infinite. On the plate at x =d assume a sheet current density of $\mathbf{K} = -K\hat{\mathbf{a}}_z \frac{A}{m}$ and on the plate at x = 0 $\mathbf{K} = K\hat{\mathbf{a}}_z \frac{A}{m}$. Assume free space between the plates.




This is similiar to the solenoid we discussed in class in that the H-field is very weak outside the plates and can be ignored compared to the field between the plates.

calculate magnetic flux, $\Psi$, through this cross-section

$$\oint \vec{H} \cdot d\vec{l} = H_{inside} W = I_{enclosed} = KW$$

$$\vec{H}_{inside} = K\hat{a}_y$$

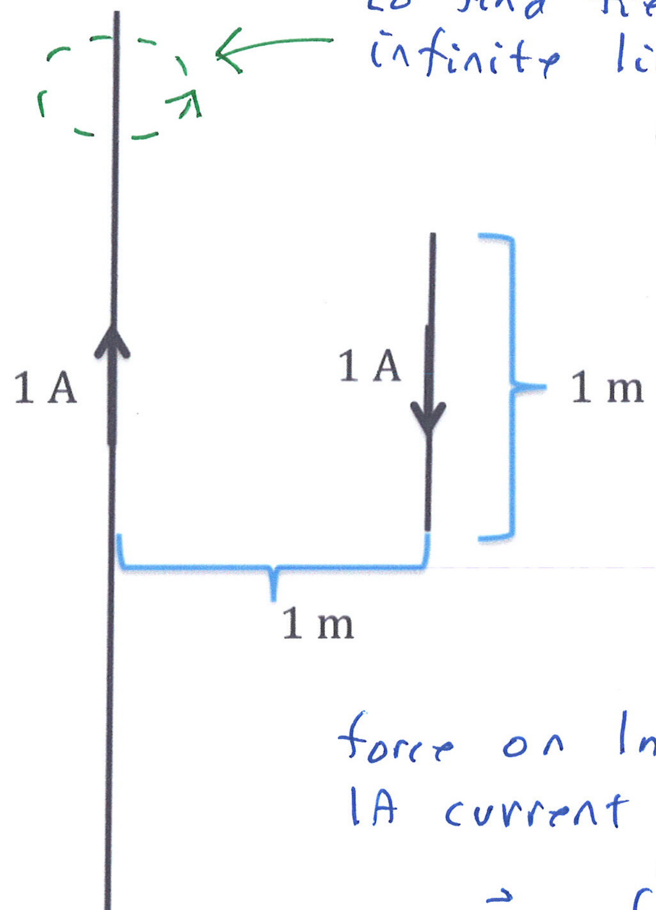
$$\vec{B}_{inside} = \mu_0 K \hat{a}_y$$

$$\Psi = B_{inside} \ell d = \mu_0 K \ell d$$

$$L = \frac{\Psi}{I} = \frac{\mu_0 K \ell}{KW} = \frac{\mu_0 \ell d}{W}$$

$$\frac{L}{\ell} = \frac{\mu_0 d}{W} = \text{inductance per unit length}$$

(12pts). 5. An infinitely long striaght wire has a current flowing through it of 1 A. Parallel to this infinite line is a 1 m long line with 1 A flowing in the opposite direction. What is the force on the 1 m line? Assume free space everywhere. Note $\mu_o = 4\pi x 10^{-7}\,\frac{H}{m}$.




to find field caused by the infinite line of current

$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed} = 1A$$

$$H_\phi 2\pi\rho = 1A$$

$$\vec{H} = \frac{1A}{2\pi\rho}\hat{a}_\phi$$

$$\vec{B} = \mu_o \frac{1A}{2\pi\rho}\hat{a}_\phi$$

force on 1m 1A current $= \vec{F} = \int I\vec{dl}\times\vec{B}$

$$\vec{F} = \int_{1m}^{0} 1A\, dz\hat{a}_z \times \mu_o \frac{1A}{2\pi(1m)}\hat{a}_\phi$$

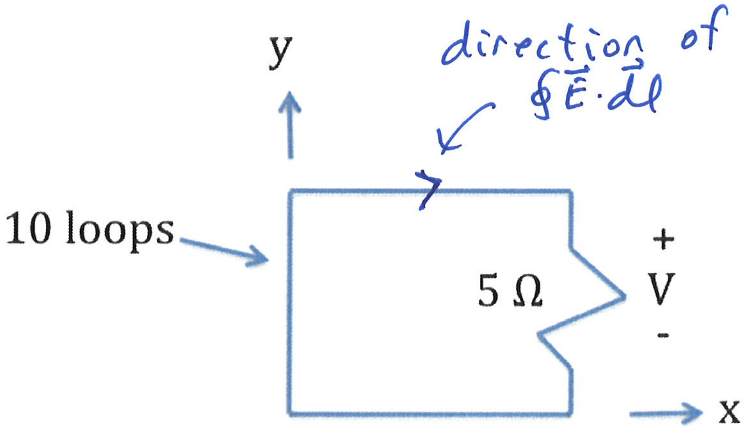
$$\vec{F} = \frac{(1A^2)\mu_o}{2\pi m}\left[\int_{1m}^{0} dz\right](-\hat{a}_\rho) = \frac{(1A^2)\mu_o}{2\pi m}(-1m)(-\hat{a}_\rho)$$

$$= \frac{1A^2}{2\pi m}\left(4\pi\times10^{-7}\frac{H}{m}\right)(1m)\,\hat{a}_\rho = 2\times10^{-7}\,\frac{A^2H}{m}\,\hat{a}_\rho$$

$$\left[\frac{A^2H}{m} = \frac{A^2}{m}\frac{Wb}{A} = \frac{A\,Wb}{m} = \frac{AVs}{m} = \frac{C}{s}\frac{J}{C}\frac{s}{m} = \frac{J}{m} = N\right]$$

$$\vec{F} = 2\times10^{-7}\,\hat{a}_\rho\ N$$

(12pts) 6. Shown is a wire shaped into a 10 turn square loop of 1m x 1m. A $5\Omega$ resistor is connected between the ends of the wire. The loop is in the xy plane. The magnetic flux density is $\mathbf{B} = 0.01\sin(1000t)\,\hat{\mathbf{a}}_z \frac{\text{Wb}}{\text{m}^2}$. What is the voltage V?




$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\lambda}{dt}$$

$$\Psi_{\text{into page}} = -(0.01)s$$

$$\Psi_{\text{into page}} = -\left[(0.01)\sin(1000t)\,\frac{\text{Wb}}{\text{m}^2}\right] 1\,\text{m}^2 = -(0.01)\sin(1000t)\ \text{Wb}$$

$$\frac{d\Psi}{dt}_{\text{into page}} = -(0.01)(1000)\cos(1000t)\,\frac{\text{Wb}}{\text{s}} = -10\cos(1000t)\ \text{V}$$

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt}_{\text{into page}} = (-10)(-10\cos(1000t))\ \text{V}$$

$$V_{emf} = 100\cos(1000t)\ \text{V} = V$$

(12 pts) 7. A conducting rod lays on two rails as shown. It is pushed with a force so that it moves with constant velocity $\mathbf{u}=5\hat{\mathbf{a}}_x \frac{\text{m}}{\text{s}}$ and everywhere the magnetic flux density is $\mathbf{B} = -2\hat{\mathbf{a}}_z \text{ T}$.


direction of induced current from Lenz's law

(6 pts) A) What is the voltage V?

$\Psi_{\text{into page}} = BA = \left(2\frac{\text{Wb}}{\text{m}^2}\right)(1\text{m})(l-x)$ where $l$ = length of rails, $x$ = position of rod

$\frac{d\Psi}{dt}_{\text{into page}} = \left(2\frac{\text{Wb}}{\text{m}^2}\right)\left(-\frac{dx}{dt}\right) = \left(2\frac{\text{Wb}}{\text{m}^2}\right)\left(-5\frac{\text{m}}{\text{s}}\right) = -10\text{V}$

$V_{emf} = V = \underbrace{\oint \vec{E}\cdot d\vec{l}}_{\text{clockwise around circuit}} = -\frac{d\Psi}{dt}_{\text{into page}} = 10\text{V}$

$i = \frac{10\text{V}}{10\Omega} = 1\text{A}$

(6 pts) B) What is the force F?

$\vec{F}_m = \int I d\vec{l} \times \vec{B} = \int_0^{1\text{m}} (1\text{A})\, dy\, \hat{a}_y \times \left(-2\hat{a}_z \frac{\text{Wb}}{\text{m}^2}\right) = -2\frac{\text{A Wb}}{\text{m}^2}\int_0^{1\text{m}} dy\, \hat{a}_x$

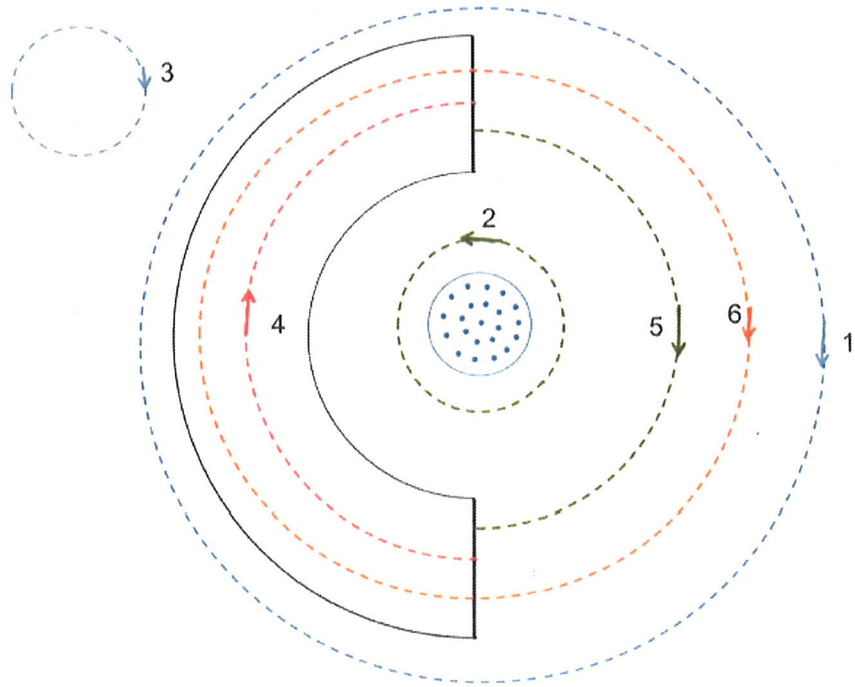
$= \left(-2\frac{\text{A Wb}}{\text{m}^2}\right)(1\text{m})\,\hat{a}_x$

$\left[\frac{\text{A Wb}}{\text{m}} = \frac{\text{C}}{\text{s}}\frac{\text{V s}}{\text{m}} = \text{C}\frac{\text{J}}{\text{C}}\frac{1}{\text{m}} = \frac{\text{J}}{\text{m}} = \text{N}\right]$

$\vec{F}_m = -2\hat{a}_x \text{ N}$

So the force that has to be applied to move the rod at $u = 5\hat{a}_x \frac{\text{m}}{\text{s}}$ $= \vec{F} = -\vec{F}_m = 2\hat{a}_x \text{ N}$

(12 pts) 8. Shown are a conductor in the form of a half-ring and a solenoid. The conducting half-ring is in the plane of the page and the solenoid is through the page. The dots in the center of the solenoid indicate the direction the magnetic flux is increasing due to an increasing current in the solenoid resulting in a rate of change of magnetic flux out-of-the page of $\frac{d\phi}{dt} = 10\,V$. The dashed lines indicate paths in the plane of the page. For the paths shown find the following,




$\oint_1 \mathbf{E} \bullet \mathbf{dl} =$ 10 V

$\oint_2 \mathbf{E} \bullet \mathbf{dl} =$ - 10 V

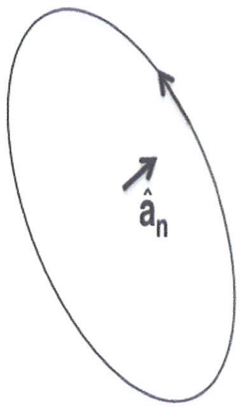
$\oint_3 \mathbf{E} \bullet \mathbf{dl} =$ 0

$\int_4 \mathbf{E} \bullet \mathbf{dl} =$ 0

$\int_5 \mathbf{E} \bullet \mathbf{dl} =$ 10 V

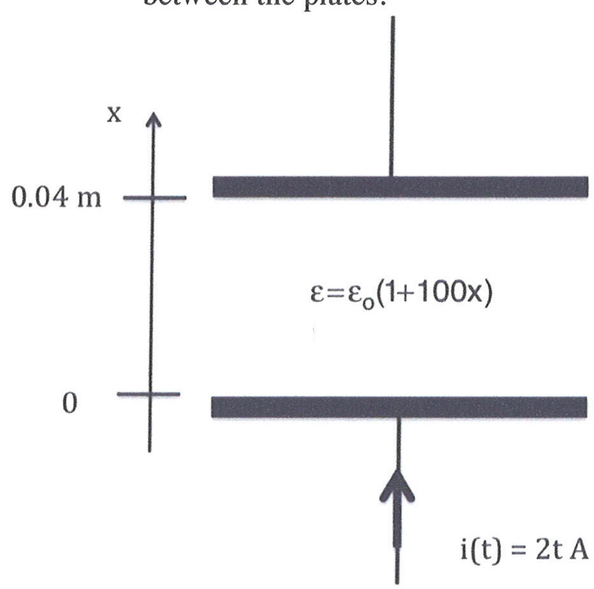
$\oint_6 \mathbf{E} \bullet \mathbf{dl} =$ 10 V

(6 pts) 9. Shown is a circular conducting ring with current 1A flowing around it as shown. The area of the ring is 0.01 $m^2$. The ring is placed in the uniform magnetic field $\mathbf{B} = 0.1\hat{\mathbf{a}}_z$ T. If the ring is free to move, in what direction will $\hat{\mathbf{a}}_n$ point?




$\hat{a}_z$

(6 pts) 10. A current, i(t) = 2t A, is charging the parallel plate capacitor shown. The plate separation is 0.04 m and the plate area $1m^2$. Determine the displacement current density everywhere between the plates.




$$J_D = \frac{\partial D}{\partial t} = \frac{\partial \rho_s}{\partial t}$$

$$= \frac{1}{A}\frac{\partial Q}{dt}$$

$$= \frac{1}{A} i(t)$$

$$= \frac{2t\,A}{1\,m^2} = 2t\,\frac{A}{m^2}$$

$$\vec{J}_D = 2t\,\hat{a}_x\,\frac{A}{m^2}$$